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Preliminary version

Abstract

The regionalization of national input-output tables is a major issue in regional science as corresponding regional data is often unavailable. In this paper, a framework is developed to estimate intra- and interregional input-output tables. The intraregional estimates are based on the well-accepted FLQ method. The interregional estimates include gravity model-based estimates to account for geographical distances between the regions. The estimates are embedded in an interregional accounting framework which ensures consistency of regional values with the national aggregates. The framework is applied to the German economy of 2010. We are able to show the importance of taking into account interregional input-output relations in the derivation of (regional) demand multipliers.

1 Introduction

Input-output (I-O) analysis is an important tool for economic impact analysis (e.g. Hallegatte [9] 2012; West [21] 1995). However, the applicability is often limited to the national scale, because regional input-output tables are not available. Regarding European countries, official input-output tables are usually provided on the national level only. Regional input-output tables have to be constructed by applying various regionalization techniques.

Regionalization can be achieved by applying location quotients (e.g. Flegg/Webber [8] 2000). These are well-accepted tools but have the major shortcoming that they are designed to construct input-output tables for a single region and when applied to all regions (of a nation), do not necessarily yield regional input-output tables that are consistent with the national table on aggregate.
Producing consistent estimates while maintaining the advantages of location quotients will be the main concern of the interregional input-output framework developed in this paper. The consistency is ensured by using a variation of the interregional input-output (IRIO) framework presented in Canning/Wang [3] (2004). The advantages of location quotients are exploited by using the FLQ method (Flegg/Webber [7] 1997) to derive initial estimates for the intraregional input-output transactions.
The second concern are interregional intermediate transactions. A usual procedure in multi-regional accounting is to estimate interregional intermediate flows through relative sizes of sectors in different regions (cf. Batten [1] 1982). In our model, distances between regions are also accounted for in the derivation of these estimates. The dependence of trade volumes on the geographical distance is estimated on the international (EU28) level by means of a gravity model and is carried over to the intranational level.
Furthermore, we apply the IRIOLQ framework to the German economy of 2010 in a resolution of 16 regions (federal states) and 7 sectors, thereby respecting geographical distances between the regions. Our main finding is that demand multipliers derived from our interregional model are significantly higher compared to those obtained from a single-region model.

The paper is organized as follows. In chapter 2, the location quotient-based interregional input-output (IRIOLQ) framework is presented and formulated as a constrained optimization problem. The third chapter is concerned with the application of the framework for Germany. Chapter four treats the use of the framework for economic impact analysis. The final chapter concludes.

2 The IRIOLQ framework

Our starting point is the 'modeling framework to estimate interregional trade patterns and input-output accounts' (Canning/Wang [3] 2004). In fact, they present two model types. The first is called multi-regional input-output (MRIO) account. Within this framework, the interregional trade of each sector is only estimated on aggregate, not assigning the precise use as intermediate input to a specific sector (Canning/Wang [3] 2004). The second, the interregional input-output (IRIO) account, takes into account mutual intermediate flows between all sectors in all regions. Our precise model setup is an advancement of the IRIO framework, as we incorporate location quotients (LQ).

We stick to the notation of Canning/Wang [3] (2004). z_{ij}^{sr} denotes (the value of) intermediate inputs from sector i in region s to sector j in region r, m_i^r imported inputs (from outside the nation), e_i^r exports (out of the nation), v_i^r value added, x_i^r output and y_i^r domestic final demand. The corresponding variables without upper indices, z_{ij}, m_i, e_i, v_i, x_i and y_i denote national aggregates. These national values are assumed to be known with certainty. Usually, they can be obtained from a national input-output table.

2.1 Equations

The idea is to formulate a system of IRIO equations that guarantees (a) that the regional input-output tables are consistent inside each region and (b) that the regional values are consistent with the national aggregates. Concerning (a), the two relevant constraints are that both the sum over all input types and the sum over all output types have to be equal to the output of each sector in each region. The input constraint is

$$\sum_{j,s} z_{ji}^{sr} + m_i^r + v_i^r = x_i^r \qquad \text{for all } i, r. \tag{1}$$

In other words, the output value of all sectors in all regions has to be equal to the sum of received intermediate inputs, imported inputs and value added. Note that taxes and subsidies are not considered in this framework and therefore do not appear in the equation.
The output constraint is

$$\sum_{j,s} z_{ij}^{rs} + y_i^r + e_i^r = x_i^r \qquad \text{for all } i, r. \tag{2}$$

This just means that the output can be used as intermediate input, it can be exported or used to satisfy the domestic (in the sense of the nation) final demand.
Concerning (b), the relevant constraints are just aggregation constraints for output, final demand, exports and imports. In our framework, the regional valued added of each sector v_i^r is assumed to be known with certainty and therefore, the corresponding equation can be dropped. The other equations require for all i, j that

$$\sum_{s,r} z_{ij}^{sr} = z_{ij} \tag{3}$$

$$\sum_{r} x_i^r = x_i \tag{4}$$

$$\sum_{r} y_i^r = y_i \tag{5}$$

$$\sum_{r} m_i^r = m_i \tag{6}$$

$$\sum_{r} e_i^r = e_i. \tag{7}$$

An interesting feature of this framework is that it allows to start with initial estimates for the unknown variables which, on aggregate, are inconsistent with the national input-output table. An objective function is defined that measures the distance between the final estimates (variables) and the (possibly inconsistent) initial estimates. Then this distance is minimized to obtain a solution that fulfills the constraints above and is 'as close as possible' (in the sense of the squared distance) to the initial estimates. One drawback of this method is, of course, that poor initial estimates can lead to poor final estimates.

Here, we include z_{ij}^{sr}, x_i^r, y_i^r, m_i^r, e_i^r and their initial estimates, denoted by an overbar, $\bar{z}_{ij}^{sr}$ and correspondingly, into the objective function S. It becomes

$$\begin{aligned} S = &\sum_{i,j,s,r} \frac{(z_{ij}^{sr} - \bar{z}_{ij}^{sr})^2}{w_{ij}^{sr} z_{ij}^{sr}} + \sum_{i,r} \frac{(x_i^r - \bar{x}_i^r)^2}{x_i^r} + \sum_{i,r} \frac{(y_i^r - \bar{y}_i^r)^2}{y_i^r} \\ &+ \sum_{i,r} \frac{(m_i^r - \bar{m}_i^r)^2}{m_i^r} + \sum_{i,r} \frac{(e_i^r - \bar{e}_i^r)^2}{e_i^r}, \end{aligned} \tag{8}$$

where $w_{ij}^{sr} (> 0)$ denote weights which will be used later in the application to account for the fact that some estimates are more reliable than others. As an alternative to minimizing the squared distance, maximizing the entropy (Canning/Wang [3] (2004); Batten [1] (1982)) is also a possible procedure, but this is not discussed further at this point.
So far, the model presented does not differ much from that in Canning/Wang [3] (2004). However, the crucial part is to obtain values for the initial estimates of the intermediate transactions which will be done next.

2.2 Deriving initial estimates

In contrast to the experiments in Canning/Wang [3] (2004), we use the FLQ method (Flegg/Webber [7] 1997) to derive initial estimates for the *intra*-regional intermediate transactions, as the FLQ method serves to regionalize an input-output table to a single region. The FLQ method is superior to simpler location quotients (e.g. Kowalewski [12], 2013) as it accounts for the ability of sectors to produce intermediate inputs for the region in which they operate, depending on the size of the region. In the following, we present the method step by step. We construct the location quotients with the help of employment data on a regional and national level, which is a common method (Flegg/Webber [8], 2000). Employment in sector i in region r is denoted by ϵ_i^r, the aggregate employment of sector i over all regions by ϵ_i, the aggregate employment of region r over all sectors by ϵ^r and the total national employment by ϵ. Then, the simple location quotient LQ_i^r (e.g. Spoerri et al. [16] 2007; Kowalewski [12] 2013) is defined by

$$LQ_i^r = \frac{\epsilon_i^r/\epsilon^r}{\epsilon_i/\epsilon}. \tag{9}$$

Thus, LQ_i^r measures the ratio between the relative size of sector i (in terms of employment) in region r and the relative size of sector i on the national level. However, when regionalizing input-output tables, we want to account for relative sizes of sending and receiving industries. Therefore, the following cross-industry location quotient (CILQ) is defined (e.g. Kowalewski [12] 2013):

$$CILQ_{ij}^r = \frac{LQ_i^r}{LQ_j^r} \quad \Big(= \frac{\epsilon_i^r/\epsilon_i}{\epsilon_j^r/\epsilon_j} \Big). \tag{10}$$

Since by construction, $CILQ_{ij}^r = 1$ for $i = j$, one needs to adjust the 'diagonal' elements by setting $CILQ_{ii}^r = LQ_i^r$ (Smith/Morrison [15] 1974). Now, we obtain Flegg's location quotient (Flegg/Webber [7] 1997) FLQ_{ij}^r by:

$$FLQ_{ij}^r = CILQ_{ij}^r \cdot \lambda^r \tag{11}$$

$$\text{with } \lambda^r = [\log_2(1 + \epsilon^r/\epsilon)]^\delta. \tag{12}$$

The interpretation of λ^r is that the cross-industry location quotients are adjusted downward ($\lambda^r \leq 1$) to obtain Flegg's location quotient. Later in derivation of the actual estimate, this will reflect that regional industries are assumed to be less able to produce intermediate inputs for their own region than the industries on a national level. This is the core assumption of Flegg's location quotient. There exists an 'augmented' FLQ (Flegg/Tohmo [6] 2011), which allows making the assumption that cross-industry location quotients are adjusted upward. However, in empirical applications, the augmented FLQ does not necessarily perform better than the 'classical' FLQ (Kowalewski [12] 2013) and thus we decide to use the latter.

The level of adjustment in the (classical) FLQ formula (equation 11) is stronger in regions that are relatively small (in terms of employment), reflecting the assumption that industries in smaller regions need to import more of their intermediate inputs from outside the region than industries in bigger regions. The parameter δ ($0 \leq \delta \leq 1$) determines the general amount of adjustment. Appropriate values for δ are assumed to range between $\delta = 0.15$ and $\delta = 0.35$, which may differ across sectors and regions (Kowalewski [12] 2013; Flegg/Tohmo [6] 2011).

Taking into account the national input-output relations, we can now construct the FLQ-estimates for the intraregional intermediate transactions, $\bar{z}_{ij}^{rr}$. They are given by

$$\bar{z}_{ij}^{rr} = \begin{cases} \frac{z_{ij}}{x_j} \cdot \bar{x}_j^r & \text{if } FLQ_{ij}^r \geq 1 \\ FLQ_{ij}^r \cdot \frac{z_{ij}}{x_j} \cdot \bar{x}_j^r & \text{if } FLQ_{ij}^r < 1. \end{cases} \tag{13}$$

Note that the original method (Flegg/Webber [7] 1997) proposed estimates for the input-output coefficients (share of intermediate inputs in output). In that setting, the formula states that the initial estimates of the regional input-output coefficients correspond to the national coefficients if the FLQ is greater than 1 and are adjusted otherwise. Our formula 13 contains $\bar{x}_i^r$, an estimate for the regional sectoral output. It is derived by assuming that each sector has a constant value added to output ratio in each region, namely the respective national ratio. Formally, this writes

$$\bar{x}_i^r = \frac{x_i}{v_i} \cdot v_i^r. \tag{14}$$

Now, we still need the initial estimates for the interregional intermediate transactions, $\bar{z}_{ij}^{sr}, s \neq r$. We use estimates that are consistent with the aggregate, meaning $\sum_{s,r} \bar{z}_{ij}^{sr} = z_{ij}$. We set

$$\bar{z}_{ij}^{sr} = g_{ij}^{sr} \cdot (z_{ij} - \sum_{r'} \bar{z}_{ij}^{r'r'}) \text{ for } \quad s \neq r, \tag{15}$$

with g_{ij}^{sr} being a parameter that has to be constructed in a two step procedure. For now, we use the simple approach of estimating interregional intermediate transactions by relative sizes of sectors in sending and receiving region (cf. Batten [1] 1982). We define

$$h_{ij}^{sr} = \begin{cases} \bar{x}_i^s \bar{x}_j^r & \text{for } \quad s \neq r \\ 0 & \text{for } \quad s = r. \end{cases} \tag{16}$$

This allows us to construct consistent estimates easily by setting

$$g_{ij}^{sr} = \frac{h_{ij}^{sr}}{\sum_{s,'r'} h_{ij}^{s'r'}} \text{ for } \quad s \neq r. \tag{17}$$

Now, one can verify that

$$\begin{aligned} \sum_{s,r} \bar{z}_{ij}^{sr} &= \sum_{\substack{s,r \\ s \neq r}} \bar{z}_{ij}^{sr} + \sum_{\substack{s,r \\ s = r}} \bar{z}_{ij}^{sr} \\ &= \sum_{\substack{s,r \\ s \neq r}} g_{ij}^{sr} \cdot (z_{ij} - \sum_{r'} \bar{z}_{ij}^{r'r'}) + \sum_{r'} \bar{z}_{ij}^{r'r'} \\ &= (z_{ij} - \sum_{r'} \bar{z}_{ij}^{r'r'}) \cdot \sum_{\substack{s,r \\ s \neq r}} \frac{h_{ij}^{sr}}{\sum_{s,'r'} h_{ij}^{s'r'}} + \sum_{r'} \bar{z}_{ij}^{r'r'} \\ &= z_{ij}. \end{aligned} \tag{18}$$

Finally, we only have to construct initial estimates for final demand y_i^r, imports m_i^r and exports e_i^r. Regarding the estimate for the final demands, $\bar{y}_i^r$, the national sectoral final demand is split onto the regions by using the information on regional value added:

$$\bar{y}_i^r = y_i \cdot \frac{v_i^r}{v_i}. \tag{19}$$

Import and export estimates are derived from foreign trade data. We assume that the shares of the regions in the national imports and exports are known and they are denoted by $imsh^r$ and $exsh^r$. We obtain as estimate for the regional imports

$$\bar{m}_i^r = m_i \cdot imsh^r \tag{20}$$

and for the regional exports

$$\bar{e}_i^r = e_i \cdot exsh^r. \tag{21}$$

This completes the construction of the initial estimates. Note that the regional data we use is that on sectoral value added, sectoral employment and aggregate shares of imports and exports. The procedure to obtain the final estimates is to minimize S subject to equations 1 to 7.

2.3 Adding distances to interregional transaction estimates

A major problem in the approach as presented so far is that it does not account for distance when estimating the interregional intermediate flows. Estimates for inter-industry trade between different regions only depend on the sizes of the sectors in the considered regions. However, the framework above allows to incorporate distance estimates. These do not have to represent geographical distance only.

The determination of the extent to which certain factors like distance influence trade volumes is the idea behind gravity models (Tinbergen [19] 1962; Linneman [13] 1966). Regarding interregional trade, parameters estimated from gravity-type models at an international level can be used to forecast regional trade (Fingleton et al. [5] 2014; Riddington et al. [14] 2006).

Let us assume that trade data is available on a national level. The following gravity model can be estimated:

$$\ln t^{qp} = \alpha + \beta \ln d^{qp} + \sum_{k=1}^{K} \gamma_k \ln u_k^{qp} + \eta^{qp}, \tag{22}$$

where t^{qp} denotes the (monetary) trade volume between the sending country q and the receiving country p. The distance between country q and p is given by d^{qp}. The u_k^{qp} are control variables with coefficients γ_k. The η^{qp} are error terms.

The estimated parameter $\bar{\beta}$ is used to construct estimates for the trade of sectors between regions. The estimates h_{ij}^{sr} in equation 16 are updated to include distances and become

$$\hat{h}_{ij}^{sr} = \bar{x}_i^s \cdot \bar{x}_i^r \cdot (d^{sr})^{\bar{\beta}} \text{ for } s \neq r. \tag{23}$$

Note that $\bar{\beta}$ shows up as an exponent because the gravity equation is estimated for logarithmized data.

The combination of gravity models and location quotient methods in the derivation of interregional trade and input-output relations should provide an opportunity for further research. In particular, sectoral gravity models which would allow us to derive sector-specific estimates of the dependence of trade on distance.

3 Applying the IRIOLQ framework

In this section, we apply the IRIOLQ framework to the German economy of 2010. First of all, we want to discuss the issue of data availability. Table 1 shows the data used and its sources.

Since the national input-output table is the major source of information and is not published annually, we have to use the table from 2010. This implies that also the other data should come from 2010.
The sectoral structure of the input-output table, and therefore that of the IRIOLQ framework, is based on the CPA2008 classification (Classification of Products by Activity). This classification is useful for input-output analysis because economic activity of a single firm is split onto sectors according to the actual composition of its activity. Unfortunately, this method does not coincide with the German WZ2008 classification, where the whole activity of a single firm is always attributed to a single sector. The WZ2008 classification is the underlying structure of the regional employment and value added data. The definition of economic activities belonging to the sectors is similar in both classifications and we use the following 7 sectors.
Sector A includes agriculture, forestry and fishing, sector BC mining and quarrying; manufacturing, sector DE electricity, gas, steam and air conditioning supply; water supply; sewerage, waste management and remediation activities, sector F construction, sector GJ wholesale and retail trade; repair of motor vehicles and motorcycles; transportation and storage; accommodation and food service activities; information and communication, sector KN financial and insurance activities; real estate activities; professional, scientific and technical activities; administrative and support service activities, sector OT public administration and defense; compulsory social security; education; human health and social work activities; arts, entertainment and recreation; other service activities; activities of households as employers.
The regions correspond to the 16 federal states of Germany, Baden-Württemberg (BW), Bavaria (BY), Berlin (BER), Brandenburg (BRB), Hamburg (HH), Bremen (HB), Hesse (HES), Mecklenburg-Vorpommern (MVP), Lower Saxony (NDS), North Rhine-Westphalia (NRW), Rhineland-Palatinate (RLP), Saarland (SRL), Saxony (SAX), Saxony-Anhalt (SXA), Schleswig-Holstein (SH) and Thuringia (TH).

The trade data t^{qp} for the estimation of the gravity model is taken from the Eurostat database on international trade. The control variables are GDP and population, both of sending and of receiving country. The estimated value for the influence of geographical distance on trade is $\bar{\beta} = -1.54$.

Table 1: Data and sources

Variable	Description	Source
v_i	National sectoral gross value added	Statisisches Bundesamt [17] (2010)
v_i^r	Regional sectoral gross value added	VGRdL [20] (2014)
z_{ij}	National I-O flows	Statistisches Bundesamt [17] (2010)
e_i	National sectoral exports	Statistisches Bundesamt [17] (2010)
m_i	National sectoral imports	Statistisches Bundesamt [17] (2010)
y_i	National sectoral final demand	Statistisches Bundesamt [17] (2010)
$exsh^r$	Regional share in exports	Statistisches Bundesamt [18] (2013)
$imsh^r$	Regional share in imports	Statistisches Bundesamt [18] (2013)
d^{sr}	Distance matrix between regions	own calculations
d^{qp}	Distance matrix between EU28 countries	own calculations
t^{qp}	Trade between EU28 countries	Eurostat [4]
$\epsilon, \epsilon^r, \epsilon_i, \epsilon_i^r$	Employment data	Bundesagentur für Arbeit [2] (2012)

Three problems occurred when we applied the IRIOLQ framework. First, the aggregated regional sectoral data on value added does not coincide with that from the national input-output table because they are based on different sector classifications as described above. Consequently, we have to adjust the WZ2008 data v_i^r to $({}^{v_i}/\sum_{r'} v_i^{r'})v_i^r$ to start with consistent values on the national aggregation level. A similar problem arises for the location quotients, because we use WZ2008 employment data to regionalize the input-output table based on the CPA. Resulting deviations can not be ruled out.
The second problem is that of anonymized employment data in official statistics. If a sector in a region is dominated by a few firms or only one firm, the corresponding employment data is anonymized. This can occur on the state level, but is not so relevant if one considers only few (large) sectors.
The third problem arose because there are zeros in the national I-O table of 2010 (z_{ij}). Since the estimation procedure requires positive estimates, initial estimates must not be zero. This problem can be solved by setting zero values in the initial data to small positive numbers.

Another issue worth mentioning is the role of imports. In our framework, all imports considered enter the production as intermediate inputs (see equation 1). Direct imports (i.e. imports of final goods/services) do not appear in the input-output table as no domestic sector is involved in such transactions.

Furthermore, it makes sense to re-calculate the national output $x_i := \sum_j z_{ji} + m_i + v_i$ and the national final demand $y_i := x_i - e_i - \sum_j z_{ij}$ because the values for x_i from the I-O table include taxes and subsidies which we do not consider in the IRIOLQ framework.

Regarding parameters of the model that are free to set, i.e. the weights in the objective function, w_{ij}^{sr} and the exponent in Flegg's formula, δ, we proceed as follows. We set $w_{ij}^{sr} = 0.2$ for $s = r$ and $w_{ij}^{sr} = 1$ for $s \neq r$. Note that a smaller value of w_{ij}^{sr} implies a higher contribution to the sum of squared differences and thus, the solution to a variable with smaller weights will be closer to the initial estimate. The reason for choosing this weighting scheme is that we rely much more on the intraregional estimates than on the interregional estimates as the estimation procedure for the former is more sophisticated.
The value of δ is set to $\delta = 0.2$, which is in the range of reasonable values. One opportunity for a model enhancement would be to consider region-specific and/or sector-specific values for δ which has been discussed, e.g. , in Kowalewski [12] (2013).

3.1 Selected results

We now want to present selected results. Of course, it does not make sense to list all 16 regional I-O tables here, but we give some examples. Table 2 shows the estimated regional input-output table for the city (and federal state) of Hamburg (HH). Note that, in comparison to the the national table, there is one more row ι because imported inputs split into inputs from inside the nation ($\iota_j = \sum_{i,s(s \neq HH)} z_{ij}^{s,HH}$) and inputs from outside the nation (m). Correspondingly, the table contains an additional column ω to distinguish exports to other regions in the nation ($\omega_i = \sum_{j,r(r \neq HH)} z_{ij}^{HH,r}$) from exports out of the nation (e). The upper left square matrix denotes the intermediate transactions between the respective sectors inside Hamburg. As usual, x denotes output. All values are rounded to two decimal places.

It can be seen that the regional economy of Hamburg is shaped to a large extent by the service sectors GJ and KN. The intermediate transaction balance with the rest of Germany and the trade balance with the rest of the world are both positive. For example, there are positive net exports to the rest of Germany in sector KN. This can be explained by the role of Hamburg as a center for financial services.

Table 2: Estimated regional input-output table for Hamburg 2010 [million €]

	A	BC	DE	F	GJ	KN	OT	Σ	ω	y	e	x
A	2.25	48.96	0.00	0.00	1.45	0.01	1.44		86.27	53.05	279.96	473.39
BC	20.38	2363.09	82.38	392.48	616.84	43.52	218.65		4043.62	4703.49	32746.73	45231.17
DE	5.46	523.91	193.29	7.22	215.82	36.51	123.46		887.23	1169.47	504.09	3666.46
F	3.21	80.70	37.78	108.71	126.79	266.19	132.21		852.80	3947.79	39.09	5595.27
GJ	12.64	2450.48	141.71	333.63	7980.41	668.72	1087.84		11047.60	24144.65	6549.59	54417.27
KN	7.70	1449.51	166.11	365.26	3285.51	8207.45	1579.50		8798.88	15839.24	2922.14	42621.30
OT	17.10	778.90	142.64	157.06	1159.21	422.20	1200.32		3542.92	19314.66	513.00	27248.01
Σ									29259.33	69172.36	43554.59	179252.86
ι	36.70	4704.00	464.02	694.50	9330.90	5447.61	2637.21	23314.93				
m	301.43	24162.23	1220.85	1396.76	5693.37	2433.78	2376.99	37585.40				
v	66.53	8669.40	1217.69	2139.65	26006.97	25095.29	17890.40	81085.94				
x	473.39	45231.17	3666.46	5595.27	54417.27	42621.30	27248.01	179252.86				

As a second example we consider the estimated regional input-output table for the state of Baden-Württemberg (Table 3).

Table 3: Estimated regional input-output table for Baden-Württemberg 2010 [million €]

	A	BC	DE	F	GJ	KN	OT	Σ	ω	y	e	x
A	340.17	1146.93	0.11	0.06	28.27	0.38	35.78		1400.64	1171.11	1092.20	5215.64
BC	528.56	98004.61	1470.63	6672.84	7207.32	743.10	3075.33		32269.19	46853.51	132563.80	329388.90
DE	141.81	3030.87	1892.53	51.04	1036.85	263.22	759.78		3965.44	7134.19	1993.78	20269.51
F	82.73	660.78	492.77	1652.42	861.66	2714.30	1151.14		3804.68	24746.51	145.72	36312.73
GJ	323.74	12874.21	854.43	1778.04	16349.13	3031.61	4210.68		16197.28	46877.20	24224.18	126720.49
KN	200.81	7563.46	994.17	1932.14	8382.93	17633.43	5186.01		18261.05	42942.28	12110.55	115206.83
OT	444.21	4356.87	1271.26	1074.54	5389.00	2945.72	7345.58		10344.31	78622.86	2055.86	113850.22
Σ									86242.60	248347.68	174186.08	746964.31
ι	780.64	50352.71	2550.28	4546.52	17225.72	12515.72	9141.93	97113.53				
m	753.09	59843.30	2678.89	3064.46	11824.43	4902.51	4898.38	87965.06				
v	1619.87	91555.15	8064.43	15540.67	58415.18	70456.84	78045.61	323697.76				
x	5215.64	329388.90	20269.51	36312.73	126720.49	115206.83	113850.22	746964.31				

The largest sector in Baden-Württemberg is BC, which is plausible because a lot of manufacturing, in particular by the automotive industry takes place there. The extraordinary positive (foreign) trade balance reflects the fact that a lot of the manufacturing goods are exported to other countries.

As a final example, we show the estimated intermediate transactions between Hamburg and Baden-Württemberg, thereby adding the flows of both directions (Table 4).

Table 4: Estimated intermediate transactions between Hamburg and Baden-Württemberg 2010 [million €]

	A	BC	DE	F	GJ	KN	OT
A	0.80	10.99	0.00	0.00	0.63	0.01	0.37
BC	1.80	362.83	8.35	32.71	93.43	7.86	23.80
DE	0.40	35.59	15.22	0.38	18.10	3.86	7.59
F	0.26	7.15	2.85	12.26	14.45	37.40	10.93
GJ	2.70	284.82	8.75	22.52	362.27	56.87	62.14
KN	1.37	136.24	8.59	20.72	157.08	308.17	65.45
OT	1.79	56.92	8.16	8.50	88.48	41.34	85.54

It can be seen that for sectors that are relatively large in both regions, the intermediate transactions add to several hundred million Euros. For smaller sectors, there are hardly any transactions.

This concludes the presentation of the model and we now proceed by showing the applicability of the IRIOLQ framework for the analysis of economic impacts.

3.2 Economic impact analysis

The simplest way to use input-output tables for economic impact analysis is to consider the following function that relates output (x) to final demand (y) (e.g. Kowalewski [11] 2009):

$$x = Ax + y, \tag{24}$$

where A denotes the input-output coefficient matrix. This equation has to be solved for x, yielding:

$$x = (I - A)^{-1} y \qquad \text{if } \det(I - A) \neq 0, \tag{25}$$

with I denoting the identity matrix. In many cases, it is useful to approximate the so-called Leontief inverse $(I - A)^{-1}$ by the following power series (cf. Kowalewski [11], 2009) which converges to the Leontief inverse (provided the latter exists):

$$\lim_{N\to\infty} \sum_{i=0}^{N} A^i = (I - A)^{-1} \qquad \text{if } \det(I - A) \neq 0. \tag{26}$$

The change in output Δx resulting from a change in final demand by Δy can then be computed to be

$$\Delta x = (I + A + A^2 + A^3 + \dots) \cdot \Delta y. \tag{27}$$

The power series can also be interpreted as a chain of effects (Kowalewski [11] 2009). The initial effect is $I \cdot \Delta y$ as sectors adjust their own production to the new demand. In order to do so, they have to demand inputs from other sectors $A \cdot \Delta y$. Then these sectors have to adjust their own demand for intermediate inputs and the effect is $A^2 \cdot \Delta y$ and so on.

In our setup, we have intraregional and interregional input-output relations. Changes in the final demand for goods from one region will therefore not only propagate through the respective region but also through other regions because of the interregional intermediate good flows. This makes the IRIOLQ framework a very powerful tool, e.g. to assess nationwide effects of demand changes in the regions.

At this point, we assume that the variables z_{ij}^{sr}, m_i^r, e_i^r, x_i^r and y_i^r determining the interregional input-output account are known or estimated as described previously. A possible change in final demand for goods from region r is denoted by Δy^r. We assume that demand can change simultaneously for all regions. Then, the effect on output in region s, Δx^s, is obtained by considering all intermediate flows that are triggered by the demand changes in all regions r. More precisely we get according to equation 27

$$\Delta x^s = \sum_r (I^{sr} + A^{sr} + (A^{sr})^2 + (A^{sr})^3 + \dots) \cdot \Delta y^r, \tag{28}$$

with $A^{sr} = (a_{ij}^{sr}) = (z_{ij}^{sr}/x_j^r)$ denoting the intermediate transaction coefficient matrix between region s (sending) and region r (receiving) and with I^{sr} representing an identity array which ensures that the initial effect $(I \cdot \Delta y)$ on output corresponds to the initial demand change. It is defined as

$$I^{sr} = \begin{cases} I & \text{if} \quad s = r \\ 0 & \text{else.} \end{cases} \tag{29}$$

This allows us to define an **interregional Leontief inverse**:

$$L^{sr} := I^{sr} + \lim_{N\to\infty} \sum_{k=1}^{N} (A^{sr})^k. \tag{30}$$

Example: Multipliers

The demand multiplier (type I) is defined as the output effect of a unit demand shock. Type I means that the feedback effect from output to demand via income is not included (cf. West [21] 1995). Using the definition of the interregional Leontief-inverse, the type I demand multiplier for a unit change in demand for goods from sector j from region r is

$$M_j^r = \sum_s \sum_i (L^{sr})_{ij}. \tag{31}$$

Type I demand multipliers, calculated as explained above, are shown in Table 5 for the German economy of 2010.

Table 5: Type I aggregate demand multipliers for sectors and regions

	BW	BY	BER	BRB	HH	HB	HES	MVP	NDS	NRW	RLP	SRL	SAX	SXA	SH	TH
A	1.73	1.82	1.13	1.68	1.26	1.16	1.70	1.71	1.77	1.75	1.74	1.52	1.72	1.72	1.71	1.73
BC	1.76	1.75	1.61	1.53	1.32	1.46	1.63	1.57	1.65	1.71	1.68	1.54	1.65	1.60	1.54	1.68
DE	1.63	1.63	1.62	1.58	1.39	1.49	1.61	1.55	1.60	1.68	1.61	1.52	1.61	1.58	1.54	1.60
F	1.67	1.67	1.59	1.50	1.44	1.47	1.63	1.49	1.58	1.67	1.59	1.50	1.56	1.52	1.50	1.56
GJ	1.58	1.60	1.55	1.50	1.49	1.48	1.58	1.49	1.56	1.62	1.56	1.47	1.54	1.51	1.51	1.53
KN	1.43	1.44	1.44	1.38	1.42	1.37	1.47	1.35	1.42	1.47	1.42	1.35	1.40	1.39	1.38	1.39
OT	1.35	1.36	1.31	1.29	1.30	1.28	1.36	1.26	1.33	1.37	1.33	1.27	1.30	1.30	1.29	1.30

The multiplier values range from 1.13 to 1.82. In contrast to simpler approaches, the IRIOLQ framework allows modeling interregional effects of demand changes. In order to illustrate the importance of this aspect, we also consider isolated demand multipliers in the following. There, only the output change in the region originally affected by the demand change is included. This corresponds to the multiplier value one would obtain by regionalizing the I-O table to just one region, treating intermediate inputs from other regions like imports from abroad. The isolated (type I) demand multipliers are defined as $\hat{M}_j^r = \sum_i (L^{rr})_{ij}$. These are shown for the German economy of 2010 in Table 6 and, as expected, take smaller values between 1.09 and 1.70. For sector A in the region Thuringia TH, the aggregated multiplier is around 26% bigger than the isolated one (1.73 vs. 1.37).

Table 6: Isolated type I demand multipliers for sectors and regions

	BW	BY	BER	BRB	HH	HB	HES	MVP	NDS	NRW	RLP	SRL	SAX	SXA	SH	TH
A	1.58	1.70	1.09	1.41	1.18	1.10	1.48	1.47	1.55	1.61	1.42	1.35	1.45	1.41	1.47	1.37
BC	1.60	1.62	1.42	1.37	1.22	1.25	1.43	1.42	1.49	1.59	1.40	1.30	1.47	1.40	1.38	1.42
DE	1.50	1.52	1.42	1.36	1.26	1.24	1.40	1.35	1.43	1.55	1.36	1.28	1.42	1.34	1.35	1.35
F	1.55	1.56	1.40	1.25	1.31	1.27	1.43	1.27	1.41	1.56	1.33	1.29	1.36	1.27	1.30	1.29
GJ	1.44	1.49	1.39	1.31	1.32	1.24	1.38	1.33	1.39	1.50	1.31	1.26	1.39	1.32	1.32	1.32
KN	1.32	1.36	1.32	1.25	1.29	1.19	1.32	1.25	1.30	1.38	1.24	1.19	1.29	1.24	1.25	1.24
OT	1.26	1.29	1.21	1.17	1.20	1.14	1.24	1.17	1.23	1.29	1.18	1.15	1.21	1.17	1.18	1.17

Type II multipliers include the feedback effect from output to final demand via income (cf. West [21] 1995). This effect is considered for a 2-region interregional input-output model e.g. in Hujer/Kokot [10] (2001).

When using the IRIOLQ framework to include the feedback mechanism, one should consider the following aspects. First one needs to model the relation between output change and income change. We suggest using the net value added to output ratio as an approximation for this relation. Data on net value added (gross value added minus depreciation) is most likely available in the national I-O table for all sectors.

Second, one needs to specify the final demand structure in each region. Note that y_i^r in the IRIOLQ framework denotes the final demand *for* goods from region r and not *in* region r. Therefore, it seems useful to assume that the interregional final demand structure corresponds to the interregional intermediate demand structure. Sectors/regions that need to import a lot from other regions (because they are 'small') might not be able to satisfy the local final demand either.
In a third step, one needs to derive (regional) shares of domestically spent income. As the rest of the world is not modeled, any income spent abroad does not create demand for domestic goods. This can be a reasonable simplification for the analysis of regional shocks, which is the main purpose of the IRIOLQ framework.

4 Conclusion

In this paper, a framework has been developed to construct interregional input-output tables, thereby taking into account sizes of regions, relative sizes of sectors in regions and distances between regions to produce initial estimates for intra- and interregional intermediate transactions. The intraregional estimates are based on the FLQ method (Flegg/Webber [7] 1997). For the interregional estimates, we use gravity model-based estimates to respect geographical distances. The initial estimates are adjusted during an optimization process, resulting in interregional input-output tables for all regions that are consistent with the national table on aggregate.
Regarding the construction of the initial estimates, we want to pick up the discussion about δ, the exponent in the FLQ formula. In this paper, a constant value was used, but there are indications that larger regions might require a larger δ (Flegg/Tohmo [6] 2011). Furthermore, it might also differ across sectors (Kowalewski [12] 2013). However, statements about the optimal choice of parameters, mainly δ and the weights w_{ij}^{rs} can only be made if 'true' interregional input-output tables are available. This is rarely the case and 'optimal' values derived from different regions in different nations from different time periods cannot necessarily be transferred.
The main strength of the presented IRIOLQ framework is that it allows to construct I-O models which account for interregional linkages. We showed in an application to the German economy of 2010 that including these in the calculation of demand multipliers leads to significantly higher multiplier values.
Theoretically, the same mechanisms are also present on the international scale. As these are not included in the IRIOLQ framework, the derived (type I) regional and sectoral multipliers might still be underestimated. However, also the usual critique of overestimation of demand multipliers applies because price reactions are not considered and it is likely that these would reduce the (real) multiplier effect.

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