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On the Predictive Content of Nonlinear Transformations of Lagged Autoregression Residuals and Time Series Observations

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Abstract

Although many macroeconomic time series are assumed to follow nonlinear processes, nonlinear models often do not provide better predictions than their linear counterparts. Furthermore, such models easily become very complex and difficult to estimate. The aim of this study is to investigate whether simple nonlinear extensions of autoregressive processes are able to provide more accurate forecasting results than linear models. Therefore, simple autoregressive processes are extended by means of nonlinear transformations (quadratic, cubic, trigonometric, exponential functions) of lagged time series observations and autoregression residuals. The proposed forecasting models are applied to a large set of macroeconomic and financial time series for 10 European countries. Findings suggest that such models, including nonlinear transformation of lagged autoregression residuals, are somewhat able to provide better forecasting results than simple linear models. Thus, it may be possibile to improve the forecasting accuracy of linear models by including nonlinear components.

Keywords: nonlinear models, forecasting, transformations
JEL: C22, C53, C51

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1 Introduction

Linear models are known to be simple, reliable and easy to estimate (e.g., Clements et al. 2004). Despite this, nonlinear models have received much attention in recent years due to their ability to capture well-known stylized facts of macroeconomic time series (e.g., asymmetry, chaotic behavior, clusters of outliers and periods of high and low volatility). Numerous studies have shown that macroeconomic variables like unemployment rates (e.g., Neftci 1984), exchange rates (e.g., Kräger & Kugler 1993), industrial production indices (e.g., Teräsvirta & Anderson 1992) and financial variables (e.g., Zhou 2011) are more likely to follow nonlinear processes (e.g., Yavuz & Yilanci 2012, Yoon 2010 and Franses & de Bruin 2002). Nevertheless, several authors have claimed that nonlinear models do not necessarily provide better forecasting results for these variables (e.g., Clements & Smith 2000 and Boero & Marrocu 2002).

Time series can exhibit different types of nonlinearity for various reasons. Neftci (1984), for example, investigated the asymmetric behavior of quarterly unemployment rates. Accordingly, increases in unemployment rates are much more dramatic than decreases. Furthermore, business cycle indicators, such as industrial production indices and the gross domestic product show nonlinearities due to the fact that they respond asymmetrically to output shocks, depending on the current stage of the business cycle (e.g., Brännäs & De Gooijer 1994 and Potter 1995). Bleaney & Mizen (1996) explained the nonlinear behavior of exchange rates by assuming uncertainty about the true equilibrium and showed that exchange rates exhibit greater mean-reversion as the distance from the equilibrium increases. Finally, Zhou (2011) identifies the way in which central banks conduct their monetary policies as the reason for the nonlinear behavior of interest rates.

A vast number of nonlinear models have been introduced in the literature in order to model different types of nonlinearities. Such models are, for example, the Threshold model by Tong (1978), the General Autoregressive Conditional Heteroscedasticity model by Bollerslev (1986), the Markov-Swichting model by Hamilton (1989), the Smooth Transition Autoregressive model introduced by Chan & Tong (1986) and Teräsvirta (1994), the neural network models and the bilinear model by Granger & Andersen (1978). Unfortunately, these models can easily become quite complex and difficult to estimate. Their forecasting results are strongly dependent on a number of facotrs; the current state of the time series under consideration (e.g., Teräsvirta & Anderson 1992), the number of observations available to estimate the model (e.g., Teräsvirta 2005 and Tong 1990) and/or the way in which forecasts are evaluated (e.g., Clements & Smith 2001). In their study, Diebold & Nason (1990) listed a number of reasons why nonlinear models may fail to predict more accurately: (1) the positive results of nonlinearity tests are due to outliers and/or structural breaks (2) the in-sample nonlinearity properties are not large enough to produce forecasting gains (3) the wrong nonlinear model is applied. In general, a good in-sample fit does not necessarily induce a good out-of-sample forecasting performance (Clements et al. 2004). However, several authors did find promising evidence in favor of nonlinear models (e.g., De Gooijer & Kumar 1992 and Maravall 1983). Nonetheless, as pointed out by Clements et al. (2004) the overall poor forecasting performance of nonlinear models requires further research. There is still a long way to go before simple, reliable, and easy to estimate nonlinear models are available which produce superior forecasts.

This paper tries to fill this research gap by introducing two simple and easy to estimate nonlinear models that include nonlinear transformations (quadratic, cubic, trigonometric and exponential functions) of lagged time series observations and autoregression residuals. The paper investigates whether such additional components contain important information which can be used in order to systematically improve the forecasting performance of autoregressive processes. A total of 430 monthly time series are used for 10 European countries, including unemployment rates, industrial producer indices, price indices and financial variables. The average forecasting performance of these models is appraised by means of several accuracy measures. Additionally, these models are examined for suitability for time series that are positively tested for nonlinearity. Findings suggest that nonlinear transformation of lagged autoregression residuals contain information that can be useful in forecasting macroeconomic variables. This is especially true for time series that are positively tested for nonlinearity. However, the results for the nonlinear model, including transformations of lagged time series observations, do not look promising.

The rest of this paper is organized as follows. The next section briefly describes the data used in the empirical application. Section 3 provides an overview of the methodology approach. Two simple nonlinear models, the forecasting procedure and different accuracy measures are introduced at this stage. The results are discussed in the section 4 and section 5 concludes the paper.

2 Data

The following empirical application is based on an extensive data set that includes 430 monthly time series from 10 European countries. All series can be categorized into the following five groups of variables: industrial production indices, consumer prices, producer prices, unemployment rates, and financial variables (e.g., money supply and interest rates). The majority of time series span the period between January 1996 and July 2014, amounting a total of 223 monthly observations. For a detailed description of the data set, see Table A.1 in the Appendix. To obtain stationary time series firstly log differences are taken (interest rates and unemployment rates are not in logarithms). Outliers are not removed due to the fact that this may also remove nonlinear properties from the time series (see Balke & Fomby 1994 and Dijk et al. 1999). Furthermore, only seasonally unadjusted time series are used because seasonal adjustment procedures may induce nonlinearities not otherwise present (see Ghysels et al. 1996 and Franses & Paap 1999). Hereafter, $y_{i,r,t}$ refers to a stationary and seasonally unadjusted time series, where i=1,...,10 denotes the number of the economy, r=1,...,R_i the number of the time series and t=1,...,T is the time index. R_i is the total number of time series of the economy i.

As outlined in the introduction, many macroeconomic variables exhibit nonlinearities that cannot be captured by simple linear models. Hansen (2011) argued that nonlinear models should only be used if there is convincing evidence in favor for each specific case. Similarly, Rao (1981) concludes that a nonlinear model should only be used if a time series is found to be non-Gaussian. Therefore, in order to test whether the proposed forecasting models are particularly suitable for nonlinear time series, all series are tested for nonlinearity. Following the argument of Granger (1993), a battery of nonlinearity tests are applied: the RESET test by Ramsey (1969), a modification of this RESET test by Keenan (1985), a more powerful generalization of Keenan's test by Tsay (1986), the simple nonlinearity test by McLeod & Li (1983) and the BDS test for independence by Brock et al. (1995). A time series is said to be nonlinear if at least three of these tests reject the null hypothesis of linearity. Based on this criterion, about 57 percent of all time series in the data set are nonlinear. The share of nonlinear time series varies between 44 percent for financial variables and up to 86 percent for industrial production indices.

3 Empirical application

3.1 Two simple nonlinear models

One way of developing a simple nonlinear model is to take a standard, well-known model and replace some of its linear components with nonlinear factors (see Granger 2001); this is achieved in this study. Both proposed nonlinear models extend simple autoregressive processes by including nonlinear transformations. Therefore, lagged time series values $y_{i,r,t-1}$ and lagged autoregression residuals $e_{i,r,t-1}$ are transformed by means of the following four functions: quadratic, cubic, trigonometric and exponential. Since this study considers time series models, there is no straightforward interpretation of these nonlinear transformations. The purpose of such components is merely to improve the approximation to normality. Moreover, the aim of this study is to investigate whether such components contain information that can systematically improve the forecasting performance of autoregressive processes. For this reason seldomly used transformations, like the trigonometric function, are applied in addition to well-known transformations like the quadratic function.

The *nonlinear autoregressive model* of orders j and k is given by:

$$y_{i,r,t} = \alpha + \sum_{r=1}^{R} \beta_r \cdot y_{i,r,t-p} + \sum_{s=1}^{S} \gamma_s \cdot y^{\square}_{i,r,t-q} + e_{i,r,t}, \quad t = 1, ..., T, \tag{1}$$

where r and s can vary between one and twelve. $y^{\square}_{i,r,t-q}$ represents any nonlinear transformation of lagged time series observations and T is the total number of observations.

Similarly, the *nonlinear autoregressive moving average model* of orders p and q is given by:

$$y_{i,r,t} = \alpha + \sum_{p=1}^{P} \delta_p \cdot y_{i,r,t-p} + \sum_{q=1}^{Q} \xi_q \cdot e_{i,r,t-q}^{\square} + e_{i,r,t}, \quad t = 1, ..., T. \tag{2}$$

Again, lag orders p and q can vary between one and twelve and $e_{i,r,t-q}^{\square}$ represents any nonlinear transformations of lagged autoregression residuals.

Both models most likely correspond to the bilinear model by Granger & Andersen (1978). This model simply includes product terms of time series observations and residuals. It is a natural extension of the linear $ARMA$ model and has the advantage that its structural theory is analogous to that of a linear system (e.g., Rao 1981). Furthermore, such models are able to improve the approximation to normality (see Poskitt & Tremayne 1986) and capture atypical periods of outliers (see Maravall 1983). Both models introduced in this study can easily be estimated via OLS methods. Thereby, unobservable residuals are approximated by means of a long polynomial $AR(p)$ model for the sake of simplicity (see Grillenzoni 2001). All lag orders are chosen via the Bayesian information criterion, which selects lag orders of standard autoregressive terms and nonlinear components separately.

The pseudo out-of-sample forecasting procedure is computed recursively. Accordingly, each forecast is only based on information that is actually available. After each prediction step, the lag order is chosen again and models are re-estimated. Hence, selected lag orders and estimated coefficients can vary over time. In addition, a distinction is made between rolling estimation windows of fixed size ω and expanding estimation windows. When using rolling estimation windows, one observation at the beginning of the time series is dropped in each prediction step. In contrast, the number of observations that is included for the estimation of the models increases with each prediction step when using expanding estimation window. Since the first procedure is especially suitable for time series with structural breaks (see Peseran & Timmermann 2004) and the expanding window approach can lead to more efficient estimates and lower estimation uncertainty (see Herwartz 2011) both methods are applied here.

In order to have an appropriate number of observations for the estimation and a sufficient quantity of forecast errors (see Granger 1993), the forecast horizon comprises 20 percent of each time series. Accordingly, the in-sample period spans from January 1997 to October 2010 and merges 166 observations for the majority of the time series. The out-of-sample period is from November 2010 up to July 2014 for a total of 45 observations.

3.2 Forecast evaluation

Forecasts are calculated one-, six- and twelve-months ahead using iterative multi-step forecasting methods. Unobservable observations y_{t+h} are replaced by their forecasts $\hat{y}_{t+h}$ and $\epsilon_{t+h} = y_{t+h} - \hat{y}_{t+h}$ is the corresponding h-step ahead forecast error. Due to the risk of explosive models and in order to incorporate the behavior of a human forecaster (see Stock & Watson 1998) this study follows the argument of Teräsvirta (2005) and replaces unreasonable forecasts that exceed (in absolute values) any previous observations by a no-change forecast. Such a trimming procedure is clearly preferable as a model which leads to obviously unrealistic predictions is deemed untrustworthy (see Herwartz 2011).

For a total of 32 implementations (two estimation windows, four nonlinear components, four forecasting horizons) both nonlinear models are compared to simple linear benchmark models. The benchmark model for the nonlinear autoregressive model is the AR(p) model and for the nonlinear autoregressive moving average model the ARMA(p,q) model. In the following, the benchmark model will be labeled by * and the specific nonlinear model under consideration by $^\bullet$. Five accuracy measures are computed in each prediction step:

(1) Differential of relative Mean Absolute forecast Error ($DMAE$)

$$DMAE^\bullet_{i,r} = RMAE^\bullet_{i,r} - RMAE^*_{i,r}, \tag{3}$$

where

$$RMAE^\bullet_{i,r} = \frac{1}{N_i} \sum_{t=T_1+1}^{T_2} \frac{|y_{i,r,t+h} - \hat{y}_{i,r,t+h}|}{\hat{\sigma}_{i,r,t}} \tag{4}$$

is the Relative Mean Absolute forecast Error and

$$\hat{\sigma}_{i,r,t} = \sqrt{\frac{1}{t-K} \sum_{j=1}^{t} \hat{e}_{i,r,j} \cdot \hat{e}_{i,r,j}} \tag{5}$$

the strategy- and transformation invariant estimator of the residual variance, where $\hat{e}_{i,r,t}$ is computed based on the whole set of regressors $\mathbf{X} = \{\mathbb{1}, y_{-1}, ..., y_{-p_{max}}\}$. $\mathbb{1}$ is a constant vector of ones and K the column size of the regressor matrix $\mathbf{X}$. N_i is the number of forecasting points.

(2) Differential of frequencies for Minimum absolute forecast errors ($DMIN$)

$$DMIN^\bullet_{i,r} = \frac{1}{N_i} \sum_{t=T_1+1}^{T_2} I(|\epsilon^*_{i,r,t+h}| \leq |\epsilon^\bullet_{i,r,t+h}|) - I(|\epsilon^\bullet_{i,r,t+h}| \leq |\epsilon^*_{i,r,t+h}|), \tag{6}$$

where $\epsilon_{i,r,t+h} = y_{i,r,t+h}$ - $\hat{y}_{i,r,t+h}$ is the forecast error and I($\cdot$) an indicator function.

(3) Directional Accuracy loss statistic (DA)

$$DA_{i,r} = \frac{1}{N_i} \sum_{t=T_1+1}^{T_2} I(|\tilde{da}^\bullet_{i,r,t+h}| > |\tilde{da}^*_{i,r,t+h}|) - I(|\tilde{da}^\bullet_{i,r,t+h}| < |\tilde{da}^*_{i,r,t+h}|), \tag{7}$$

with

$$\tilde{da}_{i,r,t+h} = I(y_{i,r,t+h} \cdot \hat{y}_{i,r,t+h} \geq 0) - I(y_{i,r,t+h} \cdot \bar{y}_{i,r,t+h} \geq 0) \tag{8}$$

as the directional accuracy excess over the naive forecast $\bar{y}_{i,r,t+h} = \frac{1}{T_1}\sum_{t=1}^{T_1} y_{i,r,T_1-t+1}$.

(4) Relative Mean Squared Forecast Error ($RMSFE$)

$$RMSFE_{i,r} = \frac{MSFE_{i,r}^{\bullet}}{MSFE_{i,r}^{*}} \tag{9}$$

with

$$MSFE_{i,r} = \frac{1}{N_i}\sum_{t=T_1+1}^{T_2} \epsilon_{i,r,t+h}^2. \tag{10}$$

(5) Theil's U

$$U = \frac{\sqrt{\frac{1}{N_i}\sum_{t=T_1+1}^{T_2}(y_{i,r,t+h} - \hat{y}_{i,r,t+h})^2}}{\sqrt{\frac{1}{N_i}\sum_{t=T_1+1}^{T_2} y_{i,r,t+h}^2} + \sqrt{\frac{1}{N_i}\sum_{t=T_1+1}^{T_2} \hat{y}_{i,r,t+h}^2}} \tag{11}$$

Moreover, the Diebold-Mariano (DM) test statistic for comparing predictive accuracy (see Diebold & Nason 1990) is computed.

Positive values of $DMAE$ and $DMIN$ indicate that the linear benchmark model provides better forecasting results than any nonlinear model under consideration. In contrast, negative values show that a nonlinear model provides better forecasts. The opposite is true for the DA measure. A value greater than one for the $RMSFE$ accuracy measure indicates a better forecasting performance of the linear benchmark model. Finally, the Theil's U statistic is bound between 0 and 1. The lower the value of this statistic, the greater the forecast accuracy. Although only logarithmic time series are used, the absolute forecast error ϵ_{t+h} may be scale dependent. In order to avoid problems regarding the aggregation over time series and economies, all measurements are converted into scale free statistics. The relative mean absolute error, for example, is adjusted by the modeling-invariant in-sample standard error.

With a total of 430 time series and a maximum of 45 forecasting steps for each time series, 19.350 forecast errors are available to be used for the evaluation of the forecasting performance. Thus, a reliable conclusion regarding the forecasting performance of simple nonlinear models can be made, based on this huge number of available points. In order to compare alternative forecasting models, mean group statistics are calculated (see Herwartz 2011). The average forecasting performance $\hat{g}_i^{\bullet}$ of economy i is given by:

$$\hat{g}_i^{\bullet} = \frac{1}{R_i}\sum_{r=1}^{R_i} \hat{g}_{i,r}^{\bullet}, \tag{12}$$

where $\hat{g}^{\bullet}_{i,r}$ represents any of the accuracy measures described above. The cross sectional mean group statistic is then denoted by:

$$\tilde{\triangle}^{\bullet}_{G} = \frac{1}{10}\sum_{i=1}^{10} \hat{g}^{\bullet}_{i}. \tag{13}$$

For the purpose of testing the significance of the results, the null hypothesis H_0:$\tilde{\triangle}^{\bullet}_{G} = 0$ (or H_0:$\tilde{\triangle}^{\bullet}_{G} = 1$ for the $RMSFE$ accuracy measure) is tested against the alternative hypothesis H_1:$\tilde{\triangle}^{\bullet}_{G} \neq 0$ (H_1:$\tilde{\triangle}^{\bullet}_{G} \neq 1$).

4 Results

As mentioned earlier, many macroeconomic variables are more likely to follow nonlinear processes (e.g., Yavuz & Yilanci 2012). However, results obtained in previous studies indicate that nonlinear models do not necessarily provide better forecasts for these variables. Furthermore, such models easily become complex and difficult to estimate. According to Clements & Smith (2000) and Stock & Watson (1998), linear models are generally preferable to nonlinear ones. In this study, simple linear autoregressive models are extended by including nonlinear transformations (quadratic, cubic, trigonometric and exponential functions) of lagged autoregression residuals and time series observations. Their forecasting results are compared to the predictions of simple linear benchmark models and appraised by means of several accuracy measures.

The results of this forecasting comparison using the total number of available time series is given in Table 1. As can be seen, the nonlinear autoregressive model mostly provides inferior results than a simple linear autoregressive process, regardless of the applied estimation window approach. Based on the $DMIN$ and $DMAE$ accuracy measure the linear benchmark model is clearly preferable. Both statistics are mostly significantly positive, irrespective of the transformation or forecasting horizon considered. An exception is the cubic and the trigonometric function for longer forecast horizons ($h = 6$ and $h = 12$). Regarding the $DMIN$ accuracy measure there is no significant difference between the linear and the nonlinear model for these two specifications. Overall, the nonlinear model does not only produce greater relative mean absolute forecast errors but also more frequently leads to higher absolute forecast errors. Furthermore, the null hypothesis of equal forecast accuracy (DM statistic) can not be rejected in most cases. Thus, there is no significant difference between the linear and nonlinear autoregression model based on absolute forecasts errors.

[Insert Table 1 here.]

Nevertheless, this nonlinear model more frequently predicts the correct sign, as indicated by mostly significantly positive values of the directional accuracy measure. Furthermore, the $RMSFE$ accuracy measure demonstrates that it is able to provide significantly better forecasts when the forecasts horizon is short ($h = 1$). In the majority of cases, the $RMSFE$ measure is significantly lower than one, inducing that the nonlinear autoregressive model provides slightly lower mean squared forecast errors. However, this is not the case for longer forecast horizons. Finally, the Theil's U statistic indicates that the shorter the forecast horizon, the better the forecasts are.

To summarise up, nonlinear transformations of lagged time series observations seem to contain no important information for forecasting macroeconomic variables. Regarding the different transformations of lagged autoregression residuals, it seems that the quadratic and the exponential functions lead to significantly inferior predictions. Nonlinear models based on the remaining two transformations do not provide a significantly different forecasting accuracy than linear models.

In contrast, the nonlinear autoregressive moving average model is more likely to produce superior forecasts than its linear benchmark model, especially when the rolling estimation window approach is applied. It can be observed, for example, that the $DMAE$ statistic is significantly negative for longer forecast horizons. Thus, the nonlinear model provides lower relative mean absolute forecast errors. Although Theil's U statistic indicates that forecasts of the nonlinear model are generally better if the forecast horizon is short, the nonlinear autoregressive moving average model is more likely to outperform the linear benchmark model when the forecast horizon is long ($h = 6$ and $h = 12$). Thus, linear models are more likely to produce worse forecasts with an increased forecast horizon. Using the expanding estimation window approach, mean absolute forecast errors are generally lower for the nonlinear model than for the linear one. This appears to be especially true for long-run forecasts. Nevertheless, this only applies to the trigonometric function when forecasting six months ahead. These findings look slightly different when the rolling estimation window is applied. Thereby, the nonlinear autoregressive moving average model only provides significantly lower relative mean squared forecast errors if the forecast horizon is short. Otherwise, the linear benchmark model provides significantly better forecasting results with respect to this accuracy measure.

However, when assessing directional accuracy there is no significant difference between the linear and the nonlinear model. Furthermore, the linear benchmark model is still preferable if trigonometric or exponential functions for one-step ahead forecasts are used. In this case, relative mean absolute forecast errors and the frequency of lower absolute forecast errors are generally higher for the nonlinear autoregressive model. The null hypothesis of equal forecast accuracy can usually be rejected for one-step ahead forecasts. Thus, linear models provide lower absolute forecast errors when the forecast horizon is short. No significant difference between the nonlinear and the linear specification can be found when predicting six or twelve months ahead. Overall, the nonlinear model is able to produce partially better forecasts than the linear model. In most cases, however, both models show no significant difference.

Next, in order to test whether the proposed models introduced in this study are particularly suitable for nonlinear time series, the forecasting exercise is repeated for those time series that are positively tested for nonlinearity (see section 2). The corresponding forecasting results are listed in Table 2. Again, the nonlinear autoregressive model still seems to be inferior to the linear benchmark model. Forecasts based on this subsample, however, are generally better than for the full sample as indicated by overall lower Theil's U statistics. The nonlinear specification only provides lower mean squared forecast errors if the forecast horizon is short. Overall, no significant difference between the first and the second forecasting exercise can be recognized for this model. The results look slightly better if the nonlinear autoregressive moving average model. In this case, the linear benchmark models less frequently provide significantly better forecasts. Furthermore, the nonlinear model is able to produce clearly lower mean squared forecast errors when the expanding estimation window is applied. Using the rolling estimation window approach, this is only true for short-run forecasts. Generally, the trigonometric and the exponential function exhibit the worst forecasting results.

[Insert Table 2 here.]

In summary, the findings in this study partly contradict the results from Stock & Watson (1998) and Clements & Smith (2000) who claim that nonlinear models are generally not able to give better forecasts. The results do not provide overwhelming forecasting results but suggest that the forecasting accuracy may be improved by including nonlinear transformations of lagged autoregression residuals into simple autoregressive models. This is especially true for time series that are positively tested for nonlinearity. However, similarly to the results from Boero & Marrocu (2002), these results partially depend on the applied accuracy measure. Nevertheless, nonlinear transformations of lagged time series observations do not contain important information that is helpful in forecasting. Furthermore, in contrast to the findings of Diebold & Nason (1990), this study indicates that it makes sense to test for nonlinearity. It seems that the forecasting results are more favourable to the nonlinear model if time series are used which are positively tested for nonlinearity.

5 Conclusion

Although many macroeconomic variables like unemployment rates, industrial production indices, financial variables or price indices are more likely to follow nonlinear processes, prior studies have shown that nonlinear models do not necessarily provide better forecasting results (e.g., Clements & Smith 2000). A vast number of nonlinear models have been introduced in the literature due to the fact that they are able to produce well-known stylized facts like asymmetry, chaotic behavior or clusters of outliers. However, because these models easily become quite complex and difficult to estimate there is still a long way to go before simple, reliable, and easy to estimate nonlinear models are available, which also provide better forecasts (Clements et al. 2004).

In this study two simple nonlinear models were introduced. These included nonlinear transformations (quadratic, cubic, trigonometric and exponential functions) of lagged time series values and lagged autoregression residuals into simple autoregressive processes. On the basis of an extensive data set that includes 430 time series, a pseudo-out-of-sample forecasting procedure was conducted in order to examine whether these models systematically provide better forecasts than their linear counterparts. Additionally, it has been tested whether such models are particularly suitable for macroeconomic variables that are positively tested for nonlinearity.

Results suggest that including nonlinear transformations of lagged autoregression residuals into simple autoregressive models may improve the forecasting accuracy of simple $ARMA$ models. This is especially true for time series that are positively tested for nonlinearity. However, nonlinear transformations of lagged time series observations do not contain information that is useful in forecasting macroeconomic and financial time series. These findings are not overwhelming, but provide promising evidence and an interesting direction for future research. Tasks for future work could therefore include a simulation study that can demonstrate the merits and limits of these proposed forecasting models. Furthermore, this study builds a foundation for an intensive study of the theoretical properties of the proposed models.

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A Appendix

Table A.1: Detailed data information

	Description	Information
	Industrial Production Index	
1	Industrial production index: total index	2010=100, Eurostat
2	Industrial production index: consumer goods	2010=100, Eurostat
3	Industrial production index: capital goods	2010=100, Eurostat
4	Industrial production index: durable goods	2010=100, Eurostat
5	Industrial production index: non-durable goods	2010=100, Eurostat
6	Industrial production index: intermediate goods	2010=100, Eurostat
7	Industrial production index: manufacturing	2010=100, Eurostat
8	Industrial production index: mining and quarrying	2010=100, Eurostat
	Financial Market	
9	Money supply M1	local currencies
10	Money supply M2	local currencies
11	Money supply M3	local currencies
12	Long term government Bond yield in	in %, no log, OECD
13	Share price index	2010 = 100, OECD
14	Nominal effective exchange rate: broad	2010 = 100, BIS
15	Nominal effective exchange rate: narrow	2010 = 100, BIS
16	Real effective exchange rate: broad	2010 = 100, BIS
17	Real effective exchange rate: narrow	2010 = 100, BIS
	Unemployment	
18	Total unemployment rate	in %, no log, Eurostat
19	Unemployment rate: persons under 25	in %, no log, Eurostat
20	Unemployment rate: persons between 25 and 74	in %, no log, Eurostat
21	Unemployment rate: women	in %, no log, Eurostat
22	Unemployment rate: men	in %, no log, Eurostat
	Producer price index	
23	Producer price index: total	2010 = 100, Eurostat
24	Producer price index: capital goods	2010 = 100, Eurostat
25	Producer price index: durable goods	2010 = 100, Eurostat
26	Producer price index: consumer goods	2010 = 100, Eurostat
27	Producer price index: intermediate goods	2010 = 100, Eurostat
28	Producer price index: non-durable goods	2010 = 100, Eurostat
29	Producer price index: manufacturing	2010 = 100, Eurostat
30	Producer price index: mining and quarrying	2010 = 100, Eurostat
	Consumer price index	
31	Consumer price index: all items	2005=100, Eurostat
32	Consumer price index: food and non-alcoholic beverages	2005=100, Eurostat
33	Consumer price index: alcoholic beverages, tobacco and narcotics	2005=100, Eurostat
34	Consumer price index: clothing and footwear	2005=100, Eurostat
35	Consumer price index: housing, water, electricity, gas and other fuels	2005=100, Eurostat
36	Consumer price index: furnishings, household equipment and routine household maintenance	2005=100, Eurostat
37	Consumer price index: health	2005=100, Eurostat
38	Consumer price index: transport	2005=100, Eurostat
39	Consumer price index: communications	2005=100, Eurostat
40	Consumer price index: recreation and culture	2005=100, Eurostat
41	Consumer price index: education	2005=100, Eurostat
42	Consumer price index: restaurants and hotels	2005=100, Eurostat
43	Consumer price index: miscellaneous goods and services	2005=100, Eurostat

Table 1: Full sample results

		expanding window						rolling window					
		DMIN	*DMAE*	*RMFSE*	*DA*	*DM*	*U*	*DMIN*	*DMAE*	*RMFSE*	*DA*	*DM*	*U*
Nonlinear autoregressive model													
h=1	$()^2$	1.490	0.599	0.951	0.521	-0.592	0.537	2.411	0.845	0.947	0.157	-0.834	0.539
		(0.13)	(0.01)	(0.00)	(0.07)	(0.55)		(0.01)	(0.00)	(0.00)	(0.54)	(0.41)	
	$()^3$	2.320	1.465	0.961	0.551	0.557	0.538	2.536	1.460	0.651	0.128	-0.842	0.539
		(0.01)	(0.00)	(0.00)	(0.02)	(0.58)		(0.00)	(0.00)	(0.00)	(0.64)	(0.40)	
	sin()	2.250	0.765	0.853	0.630	-0.418	0.540	2.086	0.863	0.856	0.262	-0.243	0.541
		(0.01)	(0.02)	(0.00)	(0.00)	(0.68)		(0.01)	(0.01)	(0.00)	(0.30)	(0.81)	
	exp()	1.424	0.302	1.026	0.176	2.207	0.539	2.273	0.448	0.718	-0.011	-0.282	0.540
		(0.13)	(0.20)	(0.00)	(0.52)	(0.03)		(0.01)	(0.09)	(0.00)	(0.97)	(0.78)	
h=6	$()^2$	4.866	3.608	1.041	3.635	1.924	0.600	4.827	3.686	1.052	2.052	2.132	0.599
		(0.00)	(0.00)	(0.36)	(0.00)	(0.05)		(0.00)	(0.00)	(0.00)	(0.00)	(0.03)	
	$()^3$	0.970	2.484	1.054	1.440	1.156	0.604	1.737	2.344	1.081	0.900	1.533	0.602
		(0.42)	(0.00)	(0.00)	(0.00)	(0.25)		(0.15)	(0.00)	(0.00)	(0.02)	(0.13)	
	sin()	0.843	1.169	1.000	1.488	0.022	0.604	1.907	1.759	1.180	0.798	1.116	0.604
		(0.48)	(0.00)	(0.00)	(0.99)	(0.98)		(0.11)	(0.01)	(0.00)	(0.02)	(0.26)	
	exp()	5.187	2.755	1.031	3.635	1.843	0.600	5.732	3.262	1.036	2.459	2.951	0.599
		(0.00)	(0.00)	(0.29)	(0.00)	(0.07)		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
h=12	$()^2$	5.192	2.980	0.998	3.992	0.309	0.605	5.235	3.253	1.028	2.407	0.859	0.605
		(0.00)	(0.00)	(0.83)	(0.00)	(0.76)		(0.00)	(0.00)	(0.00)	(0.00)	(0.39)	
	$()^3$	0.986	1.446	1.006	1.287	0.077	0.611	2.042	1.457	1.034	1.331	0.844	0.610
		(0.48)	(0.00)	(0.22)	(0.01)	(0.94)		(0.13)	(0.00)	(0.00)	(0.00)	(0.40)	
	sin()	1.074	1.146	1.023	1.393	1.618	0.612	2.031	1.283	1.017	1.014	1.774	0.611
		(0.44)	(0.00)	(0.00)	(0.00)	(0.11)		(0.14)	(0.00)	(0.00)	(0.01)	(0.08)	
	exp()	4.522	2.357	0.996	3.841	-0.067	0.606	5.682	2.911	1.017	2.807	2.601	0.606
		(0.00)	(0.00)	(0.51)	(0.00)	(0.95)		(0.00)	(0.00)	(0.07)	(0.00)	(0.01)	
Nonlinear autoregressive moving average model													
h=1	$()^2$	-0.748	0.197	0.922	0.016	1.864	0.538	-0.292	-0.008	0.913	0.011	1.603	0.538
		(0.40)	(0.42)	(0.00)	(0.37)	(0.06)		(0.75)	(0.97)	(0.00)	(0.41)	(0.11)	
	$()^3$	-0.217	0.452	0.920	0.013	2.177	0.539	0.087	0.103	0.912	0.037	1.896	0.538
		(0.81)	(0.09)	(0.00)	(0.58)	(0.03)		(0.92)	(0.66)	(0.00)	(0.05)	(0.06)	
	sin()	4.054	0.931	0.966	0.037	3.414	0.540	4.321	0.845	0.966	0.006	3.557	0.540
		(0.00)	(0.00)	(0.00)	(0.07)	(0.00)		(0.00)	(0.00)	(0.00)	(0.79)	(0.00)	
	exp()	3.949	0.730	0.974	0.025	3.925	0.541	3.905	0.600	0.974	0.011	4.129	0.541
		(0.00)	(0.00)	(0.00)	(0.20)	(0.00)		(0.00)	(0.01)	(0.00)	(0.59)	(0.00)	
h=6	$()^2$	0.143	0.391	1.017	-0.012	1.474	0.604	-0.184	-1.502	1.024	-0.018	0.319	0.724
		(0.89)	(0.27)	(0.00)	(0.59)	(0.14)		(0.82)	(0.00)	(0.00)	(0.41)	(0.75)	
	$()^3$	-0.389	0.205	1.000	0.006	1.286	0.604	-0.435	-1.725	1.006	-0.030	-0.452	0.724
		(0.72)	(0.59)	(1.00)	(0.82)	(0.20)		(0.60)	(0.00)	(0.04)	(0.20)	(0.65)	
	sin()	2.616	-0.412	0.990	-0.016	1.119	0.605	-0.796	-1.929	0.984	-0.008	-1.645	0.722
		(0.01)	(0.32)	(0.08)	(0.56)	(0.26)		(0.34)	(0.00)	(0.00)	(0.79)	(0.10)	
	exp()	2.893	-0.093	0.999	-0.010	1.274	0.606	-0.809	-2.420	0.984	-0.032	-1.714	0.723
		(0.00)	(0.84)	(0.89)	(0.69)	(0.20)		(0.33)	(0.00)	(0.00)	(0.21)	(0.09)	
h=12	$()^2$	2.371	0.700	0.820	-0.004	0.785	0.613	-0.578	-1.624	1.020	-0.021	-0.517	0.722
		(0.05)	(0.10)	(0.00)	(0.87)	(0.43)		(0.59)	(0.00)	(0.00)	(0.41)	(0.61)	
	$()^3$	1.809	0.396	0.840	0.021	1.047	0.613	-0.102	-2.016	1.022	-0.042	-0.158	0.723
		(0.12)	(0.24)	(0.00)	(0.51)	(0.30)		(0.92)	(0.00)	(0.00)	(0.11)	(0.87)	
	sin()	0.863	-0.008	0.845	0.011	0.869	0.614	-0.141	-1.828	1.021	-0.016	-0.417	0.722
		(0.46)	(0.98)	(0.00)	(0.69)	(0.39)		(0.89)	(0.00)	(0.00)	(0.64)	(0.68)	
	exp()	1.613	0.026	0.818	-0.003	0.107	0.614	-0.630	-2.457	1.000	-0.020	-1.551	0.722
		(0.16)	(0.95)	(0.00)	(0.92)	(0.92)		(0.55)	(0.00)	(0.98)	(0.49)	(0.12)	

Note: DMIN = Differential of frequencies for Minimum absolute forecast errors, DMAE = Differential of relative Mean Absolute forecast Error, RMSFE = Relative Mean Squared Forecast Error, DM = Diebold-Mariano test based on absolute error loss, U = Theil's U, p-values in parenthesis.

Table 2: Subsample results

		expanding window						rolling window					
		DMIN	*DMAE*	*RMFSE*	*DA*	*DM*	*U*	*DMIN*	*DMAE*	*RMFSE*	*DA*	*DM*	*U*
		Nonlinear autoregressive model											
h=1	$()^2$	2.058	0.407	0.947	0.189	-0.448	0.480	2.210	0.656	0.941	-0.198	-0.816	0.483
		(0.10)	(0.20)	(0.00)	(0.63)	(0.65)		(0.06)	(0.08)	(0.00)	(0.54)	(0.41)	
	$()^3$	1.989	1.470	0.953	0.140	0.368	0.482	2.616	1.474	0.554	-0.495	-0.927	0.484
		(0.07)	(0.00)	(0.00)	(0.68)	(0.71)		(0.01)	(0.00)	(0.00)	(0.23)	(0.35)	
	sin()	2.082	0.525	0.824	0.372	-0.252	0.483	2.035	0.800	0.827	-0.128	0.002	0.486
		(0.04)	(0.27)	(0.00)	(0.21)	(0.80)		(0.05)	(0.07)	(0.00)	(0.73)	(0.10)	
	exp()	1.933	-0.001	1.011	-0.140	2.122	0.482	2.168	0.266	0.616	-0.257	-0.671	0.484
		(0.11)	(0.99)	(0.08)	(0.71)	(0.03)		(0.06)	(0.47)	(0.00)	(0.41)	(0.50)	
h=6	$()^2$	6.247	4.595	1.042	2.991	1.680	0.556	6.256	4.593	1.064	0.858	1.943	0.556
		(0.00)	(0.00)	(0.59)	(0.03)	(0.09)		(0.00)	(0.00)	(0.00)	(0.12)	(0.05)	
	$()^3$	0.876	3.608	1.074	1.466	1.118	0.560	1.616	3.262	1.110	0.261	1.389	0.559
		(0.54)	(0.01)	(0.02)	(0.02)	(0.25)		(0.24)	(0.00)	(0.00)	(0.47)	(0.17)	
	sin()	0.526	1.538	0.986	1.633	-0.126	0.559	1.336	2.368	1.269	0.439	1.052	0.560
		(0.71)	(0.01)	(0.58)	(0.00)	(0.90)		(0.34)	(0.03)	(0.00)	(0.18)	(0.29)	
	exp()	5.633	3.580	1.010	2.897	1.382	0.556	6.573	4.100	1.018	1.511	2.795	0.557
		(0.00)	(0.00)	(0.85)	(0.04)	(0.17)		(0.00)	(0.00)	(0.09)	(0.03)	(0.01)	
h=12	$()^2$	7.585	4.335	1.018	3.303	0.823	0.566	7.665	4.492	1.068	1.145	1.386	0.567
		(0.00)	(0.00)	(0.16)	(0.02)	(0.41)		(0.00)	(0.00)	(0.00)	(0.07)	(0.17)	
	$()^3$	1.583	2.019	1.027	1.195	0.548	0.572	1.936	1.986	1.070	0.878	1.162	0.570
		(0.36)	(0.00)	(0.00)	(0.09)	(0.58)		(0.24)	(0.00)	(0.00)	(0.03)	(0.25)	
	sin()	1.447	1.575	1.026	1.356	1.363	0.572	1.485	1.716	1.019	0.631	1.403	0.571
		(0.39)	(0.00)	(0.00)	(0.02)	(0.17)		(0.36)	(0.00)	(0.00)	(0.09)	(0.16)	
	exp()	6.082	3.447	0.998	2.936	0.021	0.566	7.328	4.048	1.029	1.862	2.616	0.567
		(0.00)	(0.00)	(0.88)	(0.04)	(0.98)		(0.00)	(0.00)	(0.06)	(0.01)	(0.01)	
		Nonlinear autoregressive model											
h=1	$()^2$	-1.187	0.103	0.867	0.019	0.614	0.474	-0.371	0.043	0.863	0.037	0.478	0.475
		(0.27)	(0.75)	(0.00)	(0.41)	(0.54)		(0.73)	(0.89)	(0.00)	(0.05)	(0.63)	
	$()^3$	-0.427	0.496	0.866	0.019	0.876	0.475	0.288	0.299	0.862	0.065	0.735	0.476
		(0.70)	(0.16)	(0.00)	(0.56)	(0.38)		(0.79)	(0.41)	(0.00)	(0.02)	(0.42)	
	sin()	4.750	0.852	0.921	0.065	1.713	0.477	5.788	1.091	0.922	0.019	2.065	0.478
		(0.00)	(0.02)	(0.00)	(0.02)	(0.09)		(0.00)	(0.00)	(0.00)	(0.53)	(0.04)	
	exp()	4.926	0.748	0.930	0.046	2.373	0.477	5.529	0.748	0.924	0.019	2.334	0.478
		(0.00)	(0.03)	(0.00)	(0.10)	(0.02)		(0.00)	(0.02)	(0.00)	(0.48)	(0.02)	
h=6	$()^2$	1.266	0.434	0.976	-0.010	1.020	0.558	-0.105	-2.173	1.034	-0.021	0.896	0.723
		(0.33)	(0.31)	(0.00)	(0.74)	(0.31)		(0.92)	(0.00)	(0.00)	(0.48)	(0.37)	
	$()^3$	0.659	0.157	0.973	0.010	0.912	0.558	-0.502	-2.402	1.010	-0.031	0.179	0.723
		(0.62)	(0.75)	(0.00)	(0.76)	(0.36)		(0.62)	(0.00)	(0.01)	(0.37)	(0.86)	
	sin()	1.978	-1.078	0.956	0.021	0.286	0.558	-1.476	-2.866	0.984	0.000	-1.020	0.721
		(0.12)	(0.04)	(0.00)	(0.59)	(0.78)		(0.15)	(0.00)	(0.00)	(1.00)	(0.31)	
	exp()	2.711	-0.725	0.944	0.021	0.480	0.560	-1.088	-4.014	0.988	-0.042	-1.416	0.721
		(0.03)	(0.09)	(0.00)	(0.56)	(0.63)		(0.29)	(0.00)	(0.00)	(0.29)	(0.16)	
h=12	$()^2$	2.461	0.425	0.749	-0.037	0.439	0.569	-0.631	-2.112	1.038	-0.049	0.347	0.718
		(0.09)	(0.35)	(0.00)	(0.32)	(0.66)		(0.62)	(0.00)	(0.00)	(0.21)	(0.73)	
	$()^3$	1.818	0.425	0.766	-0.012	0.649	0.571	-0.680	-2.742	1.042	-0.074	0.508	0.719
		(0.19)	(0.41)	(0.00)	(0.78)	(0.52)		(0.59)	(0.00)	(0.00)	(0.06)	(0.61)	
	sin()	1.027	-0.813	0.768	0.037	0.104	0.569	0.532	-2.762	1.024	-0.012	-0.197	0.718
		(0.47)	(0.11)	(0.00)	(0.37)	(0.92)		(0.68)	(0.00)	(0.00)	(0.74)	(0.84)	
	exp()	1.694	-0.636	0.742	0.025	-0.372	0.570	0.173	-3.858	1.031	-0.037	-0.450	0.718
		(0.22)	(0.20)	(0.00)	(0.48)	(0.71)		(0.89)	(0.00)	(0.00)	(0.32)	(0.65)	

Note: DMIN = Differential of frequencies for Minimum absolute forecast errors, DMAE = Differential of relative Mean Absolute forecast Error, RMSFE = Relative Mean Squared Forecast Error, DM = Diebold-Mariano test based on absolute error loss, U = Theil's U, p-values in parenthesis.

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