A Zidane Clustering Theorem
– Why top players tend to play in one team and how the competitive balance can be restored

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Abstract

Empirical evidence suggests that top players often play together in one team. Based on the "O-ring theory" (Kremer 1993) a Zidane clustering theorem is derived. It is argued that the best midfielder is most efficiently allocated when combined with an ace striker, and vice versa. This implies that better teams can payer higher wages, because players are more valuable for better teams than for weaker teams. In equilibrium all teams are of homogenous quality, otherwise a reallocation would occur on the players market. Obviously, such a clustering effect negatively affects the competitive balance. It is shown that the clustering effect must be compensated by decreasing marginal revenue for sporting success in order to restore the competitive balance. This is certainly not the case in the UEFA Champions League where the prize money is exponentially increasing thus contributing significantly to the inherent monopolization in professional sports leagues.

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1. Introduction

In 2001 French soccer star Zinedine Zidane moved from Juventus Turin to Real Madrid for the record-high transfer fee of about EUR 72 million. One year later David Beckham from Manchester United followed Zidane to Real Madrid. Jointly with Luis Figo, Roberto Carlos, Ronaldo and Raúl, they formed a team consisting of the world's best soccer players in those days which were called "Los Galácticos". Clustering of players (or workers) of the same quality is a typical empirical phenomenon that can be observed in many processes where the performance of a team depends on jointly or sequentially conducted tasks. This paper provides a theory why - for a given budget constraint - a team's overall performance is maximized if all playing positions or stages of production are filled with players or workers of equal quality. The model presented in this paper is based on the "O-ring theory" developed by Kremer (1993). In equilibrium all teams end up with equal quality on each playing position, from which follows that the best team consists of all the best available players. Obviously, this has far-reaching consequences for the competitive balance in sports since quality clustering implies monopolization in team sports. The competitive balance and the design of professional sports leagues is a well-established and intensively discussed issue in sports economics (cf. Szymanski 2003, El Hodri/Qurik 1971, e.g.). "rat race" competition may lead to monopolization and strong incentives to overspend. It is often argued that competition in professional sports leagues is a specific one that needs to be regulated (cf. Sloane 1976). The European Football Association (UEFA) has recently implemented Financial Fair Play regulation in order to avoid overspending in European football (cf. UEFA 2012a) without addressing – or even adversely affecting – the root causes of market failure (cf. Vöpel 2011). This issue is discussed in greater detail below. It is shown that a stronger redistribution is needed in order to restore the competitive balance.

2. A Zidane Clustering Theorem

Success on the pitch does not only depend on the individual quality of a team's players but decisively on whether these players form a well-organized, well-functioning team. The production process in football is rather complex and consists of a sequence of different tactical und position-related tasks: defending a goal, obtaining the ball, and finally scoring. Each failure in any of these tasks can significantly reduce a team's performance. The best forward cannot always score the game-winning goal when the goalkeeper fails several times in a match.

Kremer (1993) has proposed a specific production function for such a production process. The so-called "O-ring theory" combines multiplicative quality effects (cf. Rosen 1981, 1982) with market-based matching processes (Becker 1981).[1] The underlying production function can be written as follows:

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[1] This theory is called „O-ring theory", because in 1986 a broken O-ring seal has led to the disaster of the *Challenger* space shuttle.

$$P(q_1,...,q_n) = \beta \prod_{i=1}^{n} q_i \,, \tag{1}$$

where q_i denotes the quality of player (or worker) on playing position i, β a shift parameter and P a team's overall performance, respectively. q_i can alternatively be interpreted as the probability of executing successfully the specific task on position i of a sequential production process. The parameter β contains all other factors that determine the performance of a team, e.g. the management. Since some playing positions in soccer are more important than others, the center midfielder, e.g., the production function in (1) can be generalized by weighing positions according to their specific importance:

$$P(q_1,...,q_n) = \beta \prod_{i=1}^{n} q_i^{\alpha_i} \,. \tag{1b}$$

In sports, performance is always a relative measure: A player or a team must simply be better than the opponent regardless of the absolute strength. Consequently, q_i could also denote the relative strength of a player among all available players on position i since competition in professional sports leagues can be regarded as a *Tullock competition* (cf. Tullock 1980). For a given and time-invariant pool of players, however, the absolute strength also determines the relative strength of a team.

Such a production function does not imply that it is the weakest link of a team that determines a team's overall performance, but a weak player can obviously reduce the performance to a decisive extent. If, for example, a midfielder always fails to play the ultimate pass to the forward, i.e. $q_M = 0$, then there will be no goal regardless of the quality of the forward, i.e. $q_M \cdot q_F = 0$ for $q_M = 0$ and $q_F \in [0;1]$. For this reason, it turns out – as will be shown below – that it is efficient to hire players of the same or at least similar quality rather than of different quality.[2]

The marginal productivity (or the marginal contribution to a team's performance) is given as

$$\frac{\partial P(q_1,...,q_n)}{\partial q_j} = \beta \prod_{i \neq j}^{n-1} q_i > 0 \,. \tag{2}$$

Obviously, the marginal productivity depends positively on the quality of the teammates. This means that the cross derivative is also positive:

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[2] A player can obviously not easily be summarized in a single number. Besides the pure quality other abilities or characteristics play a crucial role for assessing a player and his potential meaning for a team.

$$\frac{\partial^2 P(q_1,...q_n)}{\partial q_j \partial q_k} = \beta \prod_{i \neq j,k}^{n} q_i > 0. \tag{3}$$

Given such a production function, factor demand, i.e. demand for players, can be derived from profit maximization. It is assumed that clubs' primary objective is to maximize success on the pitch. If there is a competitive market profits are zero in a long-run equilibrium, so that maximizing success is equivalent to profit maximization.[3]

$$\max_{\{q_1,...q_n\}} G = \beta \prod_{i=1}^{n} q_i - \sum_{i=1}^{n} w(q_i), \tag{4}$$

The first-order condition for a profit maximum is given as:

$$\frac{\partial w(q_j)}{\partial q_j} = \beta \prod_{i \neq j}^{n} q_i. \tag{5}$$

This implies that the marginal revenue from an increase in the team's performance equals the marginal wage that is paid for a marginal increase in the quality on a playing position. Since the cross derivative is positive,

$$\frac{\partial^2 P(q_1,...q_n)}{\partial q_j \partial (\prod_{i \neq j}^{n} q_i)} = \beta > 0, \tag{3b}$$

it follows directly that a team with a higher average quality is able to pay a higher wage. Note that also a better management captured by the parameter β makes players more valuable.

As long as there are teams with heterogeneous qualities, a reallocation of players will occur. Since those Teams with initially higher average quality can ay higher wages the process ends up in an equilibrium Where all Teams are of homogenous quality. Therefore, in equilibrium teams are of the same quality on each position:

$$P(q) = \beta q^n. \tag{6}$$

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[3] There is a broad discussion in sports economic literature on the primary objective of a professional sports club (cf. Vrooman 1995, e.g.). It could be profit maximization or maximization of sporting success. Under specific conditions these goals might turn out to be equivalent.

This can easily be shown for just two playing positions and two types of players, a low-quality and a high-quality player q_N, q_H, respectively. Then $q_N^2 + q_H^2 > 2 q_N q_H$ for $q_N \neq q_H$. We end up in an equilibrium with teams all of homogenous quality. To prove this let's assume that this is not the case: If q denotes the probability of successfully executing an individual task and $q_1 = 0.3$ and $q_2 = 0.8$, then the whole process or sequence of individual tasks is successfully completed with a probability of 24%. As can easily be seen, it would improve the outcome to a larger extent to become more homogenous, i.e. to strengthen the weak tie rather than the strong tie. In this simple numeric example an increase in the quality of one of the two players by 0.1 percentage points would lead to a probability of 27% in case the strong tie is strengthened and of 32% in case the weak tie is strengthened.

This is the reason, why in an equilibrium all teams are of homogenous quality. Due to equ. (5) and equ. (6), wages increase exponentially with the level of (homogenous) quality. This theoretical result is in accordance with the empirical finding that total wages tend to differ substantially between bigger and smaller clubs and superstars and mean players, respectively. The relationship between wages and quality is shown in figure 1. In a more complex model, wages would also depend on the relative importance of a playing position and the position-specific labor supply.

Figure 1: Relationship between wages and quality with clustering and homogenous teams
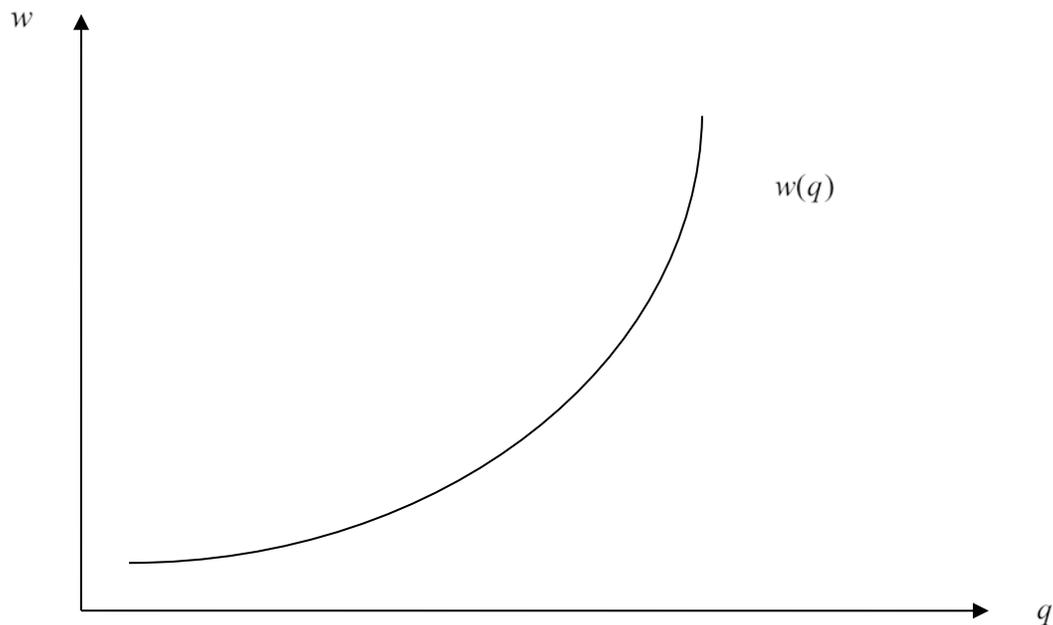
Source: own illustration.

The same result, i.e. the clustering effect of homogenous qualities, can alternatively be derived from the following two simple propositions:

MORIENTES AXIOM: *The individual performance of a player (the marginal contribution to a team's performance) depends positively on the quality of the team. A player like Morientes (forward of Real Madrid in the season 2001/02) benefits from a player like Zidane (but Real Madrid finished only in second place in the Primera Division).*

RONALDO AXIOM: *The better a player the more he benefits from an increase in the quality of the team. A player like Ronaldo benefited more from a player like Zidane than Morientes did. Real Madrid won the Spanish Championship in 2002/03 after Ronaldo replaced Morientes as the center forward.*

From these two propositions the "Zidane Clustering Theorem" can be derived:

ZIDANE CLUSTERING THEOREM: *In equilibrium players of the same quality play together in one team.*

Another explanation why homogeneity rather than heterogeneity is preferrable for teams can be derived from game theory. It can be shown in a game with mixed strategies that it is more difficult to anticipate and thus to defend the opponent's action when there are more variants of similar strenght. Somehow counter-intuitively, strengthen the strength rather than the weakness allows to play the stronger variant more often in a Nash equilibrium than the weaker one because this variant is less often defended when there is a second comparably strong option (cf. Osborne 2004, Dixit/Nalebuff 1991).

3. Implications for the competitive balance

Clustering of players with the same quality in one team has far-reaching implications for the competitive balance. In sports economics literature there is a broad discussion on how the competitive balance in professional sports leagues can be restored by regulating competition or even redistributing revenue (cf. Szymanski 2001, Késenne 2000, e.g.). It is argued that competition in sports leagues is similar to a "rat race" (cf. Akerlof 1996) implying strong short-term biased incentives, since there is an upward spiral of financial and sporting success leading to an inherent monopolization in professional sports leagues.

Such an inherent monopolization is even stronger with clustering effects that have been derived above. In the following it is shown that the distribution of revenue must exhibit certain properties in order to compensate for the clustering effect. It is assumed that the revenue depends positively on the performance depending in turn on the homogenous quality of a team, i.e. $R(P(q))$.

We assume that a team consists of a quality q_j on playing position j and of quality $\overline{q}$ on every other position. The club wants to increase the quality on position j. The marginal revenue induced by the

increase in the team's overall performance is the maximum amount of money that can be used to pay the better player. That means:

$$\frac{\partial w(q_j, \bar{q})}{\partial q_j} = \frac{\partial P(q_j, \bar{q})}{\partial q_j} \cdot R'(P)\big|_{(q_j, \bar{q})}. \tag{7}$$

As can be seen in (7) the marginal revenue depends not only on the player's own quality but also on the quality of his teammates $\bar{q}$. This implies, however, that a better team with $\tilde{q} > \bar{q}$ can pay higher wages for a better player on playing position j since he is more valuable for teammates of higher quality due to a positive externality on his teammates' performance. This would lead to a clustering of qualities in equilibrium. In order to avoid such a clustering the following conditions must hold, that means both teams can pay the same wage:

$$\frac{\partial P(q_j, \tilde{q})}{\partial q_j} R'(P)\big|_{(q_j, \tilde{q})} = \frac{\partial P(q_j, \bar{q})}{\partial q_j} \cdot R'(P)\big|_{(q_j, \bar{q})} \tag{8}$$

This condition holds if the marginal revenue of sporting success is decreasing. Since – according to the clustering effect – the same player is more valuable for the better team, i.e.

$$\frac{\partial P(q_j, \tilde{q})}{\partial q_j} > \frac{\partial P(q_j, \bar{q})}{\partial q_j} \text{ for } \tilde{q} > \bar{q},$$

it follows from (8) that

$$R'(P)\big|_{(q_j, \tilde{q})} < R'(P)\big|_{(q_j, \bar{q})}.$$

Since this must hold for any q, it follows formally:

$$\frac{\partial^2 w(q_i, \bar{q})}{\partial q_i \partial \bar{q}} = \underbrace{\frac{\partial^2 P(q_i, \bar{q})}{\partial q_i \partial \bar{q}}}_{>0} \cdot \underbrace{R'(P)\big|_{(q_i, \bar{q})}}_{>0} + \underbrace{\frac{\partial P(q_i, \bar{q})}{\partial q_i}}_{>0} \cdot \underbrace{\frac{\partial P(q_i, \bar{q})}{\partial \bar{q}}}_{>0} \cdot \underbrace{R''(P)\big|_{(q_i, \bar{q})}}_{<0} = 0 \tag{9}$$

As a result, the revenue function $R(P)$ must exhibit the following properties as a necessary (not sufficient) condition for competitive balance:

1. $R'(P) > 0$ (*"incentive compatibilty"* in terms of mechanism design theory)

2. $R''(P) < 0$ (*"competitive balance condition"*).

Otherwise, any distribution or redistribution of prize money is not effective in restoring the competitive balance. Then, a financial redistribution mechanism would only lead to a redistribution of income among players but not to a redistribution of sporting strength among clubs. Obviously, the second condition is far from being fulfilled since the prize money in the UEFA Champions League is exponentially increasing with sporting success. Moreover, qualifying for the Champions League has become crucial to stay at the top of European club football (the UEFA spent EUR 1.3 billion in 2011/12, making 17 % on average on total revenue for those clubs that have participated in the Champions League, cf. UEFA 2012b). So it is the UEFA itself that contributes significantly to the monopolization in professional football.

4. Conclusions

Clustering of players with equal quality is a phenomenon that can often be observed in team sports. Based on the "O-ring theory" (Kremer 1993) a Zidane Clustering Theorem has been derived. The underlying assumption is that players benefit from better teammates. Moreover, a player benefits the more from better teammates the better he is himself. That implies that the best midfielder is most efficiently allocated when combined with the best striker. Consequently, a better team can pay higher wages for good players than those teams with a lower quality. As a result, the competitive balance is distorted by such a clustering effect.

In order to restore the competitive balance, the marginal revenue from sporting success on the pitch should be positive (*incentive compatibility*) but must be decreasing (*competitive balance condition*) to compensate for the clustering effect. The UEFA is far from fulfilling this condition since the prize money is exponentially increasing thereby contributing significantly to the monopolization in professional football. The competitive balance is an essential and broadly discussed issue in sports economics because the "rat race" competition (cf. Akerlof 1996) in professional sports leagues provides strong short-term biased incentives and an overspending behavior both arising from the inherent monopolization.

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